than five to one. This may be partly due to the singed areas being more conspicuous but, as I have elsewhere pointed out, the disproportion between the two types of areas probably indicates also that under the influence of X-rays the yellow-white chromosome breaks more easily than the eosin singed chromosome. In the case of the flies with two areas, seven out of eight had combined areas, and this gives nine gray-singed areas to seven yellow non-singed areas, or a ratio of almost one to one, instead of five to one. Furthermore, if the X-rays happened to affect two distinct cells, in the same embryo, it is improbable that in seven out of eight cases they would lie adjacent or close to each other. For these reasons the second suggestion seems untenable.

It is impossible to study cytologically such cases as described above, so that one cannot state definitely the nature of the mechanism which produces the segregation. It almost certainly involves some form of synapsis ("pseudosynapsis") between the two X-chromosomes in the somatic cells, so that when the division occurs the two divided chromosomes of the tetrad-like figure pass to opposite poles of the spindle.

THE RADIAL VELOCITIES OF GLOBULAR CLUSTERS

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The following discussion of the radial velocities of globular clusters originated in conversations which I had with Dr. Zwicky of the California Institute of Technology during my stay at the Mount Wilson Observatory. Since it depends essentially on the validity of Strömberg's solution for the motion of the sun relative to the system of the globular clusters,[1] it will be useful to consider this solution, the more so because v. d. Pahlen and Freundlich[2] have objected to it and have concluded that the motion of the sun is probably in a direction almost at right angles to that found by Strömberg.

1. It seems strange, according to v. d. Pahlen and Freundlich, that the radial velocities of clusters, which are nearly all situated in one direction, give for the sun a large relative motion which is almost perpendicular to the direction of the clusters. They therefore assume that such a solution as Strömberg's cannot be real, and try to solve the problem by taking the simple arithmetical mean of the observed radial velocities. Since this mean value is negative, they conclude that the sun is moving toward the center of the group of clusters, and thus almost perpendicu-

larly to the direction found by Strömberg. But I think their treatment is erroneous, for the following reasons:

Let ξ, η, ζ be the components of the velocity of the sun relative to a group of celestial objects, and V_s the peculiar radial velocity of a celestial object belonging to the group. Then the observed radial velocity of the object (V) may be written in the form

$$V = a\xi + b\eta + c\zeta + V_s. \tag{1}$$

The quantities ξ, η, and ζ can always be determined by a least-squares solution, provided the values of V_s may be treated as accidental errors; that is, provided we may assume that the members of the group have no common stream motion. On the other hand, if we form the mean value of the expressions (1), we have

$$\overline{V} = \bar{a}\xi + \bar{b}\eta + \bar{c}\zeta + \overline{V}_s. \tag{2}$$

In the case of an unequal distribution of the celestial objects over the sky the mean values $\bar{a}$, $\bar{b}$, $\bar{c}$ will not in general vanish. If $\overline{V}_s \neq 0$ we have the same difficulty as with Strömberg's solution; if $\overline{V}_s = 0$ it still remains impossible to decide whether the observed negative mean value of $\overline{V}$ is caused by a motion of the sun in the direction of ξ, η, or ζ. It should be stated that v. d. Pahlen and Freundlich noticed a dependence of the distribution of positive and negative radial velocities of the clusters on galactic latitude and longitude, so that $\overline{V}_s$ may perhaps be different from zero. In this case no further investigation can be made by either method, whereas as soon as we assume (as we shall assume) that the single members of the group of globular clusters do not possess any common stream motion ($\overline{V}_s = 0$), Strömberg's solution is the only possible one.

2. From the work of Hubble[3] and Humason[4] it is known that the radial velocities of spiral nebulae show a linear correlation with the distances. The red-shift of the spectral lines increases by an amount corresponding to 500 km./sec. for an increase of 10^6 parsecs in distance. Evidently the observed radial velocities of spirals cannot be interpreted as real motions without making very artificial assumptions as to the dynamical properties of the universe. Difficulties also arise if the increase in red-shift with increasing distance is to be explained geometrically on the assumption that our world has the properties of de Sitter's solution of Einstein's equations. This possibility has been discussed by Silberstein and recently by Tolman.[5] Another suggestion, namely, that the light of very distant objects is reddened in some physical way on its long journey through space, has recently been made by Zwicky.[6] Such a physical effect will depend naturally on the density of matter in interstellar space and on the distance of the celestial object.

If the observed radial velocities of the spirals can be explained by some physical process depending on the total amount of matter between us and the spirals, then a similar red-shift should be found for globular clusters, since the much larger density inside of the galactic system compensates the much smaller distances of the globular clusters.

Since the effect depends on two variables, one the mean density between us and the celestial object, the other the distance, the empirical treatment of the problem is somewhat difficult. Fortunately, however, in the case of spiral nebulae the variations in the mean density of matter in interstellar space from nebula to nebula are small in comparison with the variations in distance. In this case, therefore, the empirical relation between *red-shift* and *distance* can be found (Hubble's correlation) by assuming a constant density of matter in interstellar space. On the other hand, for the globular clusters whose radial velocities are known the

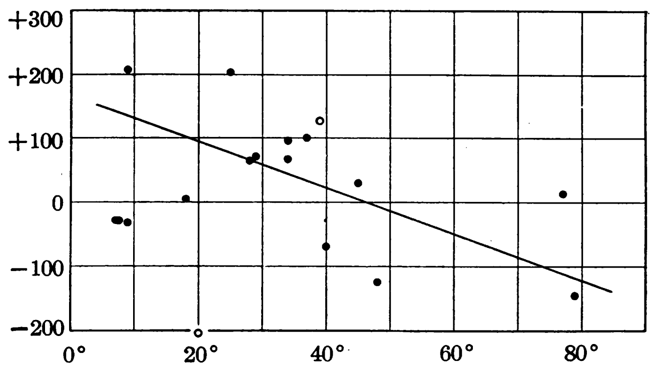

FIGURE 1

Radial velocities corrected for solar motion (ordinates) plotted against galactic latitudes.

variation in distance from cluster to cluster is small in comparison with the variation of density inside the galactic system. In this case, therefore, the empirical relation between *red-shift* and *density* can be found by assuming a constant distance for the clusters in question.

A first rough indication for the density of matter between us and a cluster will be the galactic latitude of the cluster. In figure 1 these latitudes are plotted against the radial velocities of the clusters, corrected for solar motion as found by Strömberg. The residuals resulting from Strömberg's solution show a slight dependence on galactic latitude, the red-shift being larger for clusters in small latitudes.[7] Thus, the correlation is of the kind predicted by Zwicky's explanation; nevertheless, a dynamical interpretation of this correlation cannot be ruled out *a priori*. If for instance the clusters are all falling freely toward the center of the larger galactic system in the gravitational field of its massive center, such a correlation will appear, since all the clusters whose radial velocities have been measured accurately are probably nearer than the distant center of the galactic system.

We thus have three possibilities:

(a) We interpret the correlation for the clusters on the same physical basis as that for the spirals.

(b) We interpret the correlation for the clusters dynamically. Then, however, we are left without explanation of the red-shifts shown by the spirals, because in this case neither Zwicky's hypothesis nor any other depending on the total amount of matter between us and the celestial object is valid.

(c) We interpret the correlation as a superposition of a dynamical effect and that proposed by Zwicky. Zwicky's explanation then remains valid for the spirals.

3. We discuss the first possibility a little more fully: For numerical investigations it will be much better to take as an indication of the mean density of matter between us and a cluster the number of stars inside a cone of a certain solid angle whose axis joins the sun and the cluster. Since the number of stars per square degree does not decrease *linearly* with increasing galactic latitude, we cannot expect to find a *linear* correlation between red-shift and star numbers for a cone of any arbitrarily chosen solid angle. We must therefore find a solid angle which gives a linear correlation. For our purpose it is sufficient to take the star numbers (N) from the mean distribution table of Seares and van Rhijn.[8] Star counts down to the eighteenth magnitude were used, and solutions based on the observed radial velocities (V) of the globular clusters were made in the form

$$V = a\xi + b\eta + c\zeta + \frac{N}{10^7}\,v + \mathrm{K}\,. \qquad (3)$$

ξ, η, ζ being the components of the relative velocity of the sun, v the red-shift, which is to be found, K a constant term, and N the number of stars inside the cone of given solid angle. Cones with angles of 1°, 30°, and 60° were used. Since the tabulated star numbers refer to one square degree, the numbers for 30° and 60° cones had to be derived by numerical integration.

Let β_0, λ_0 be the galactic coördinates of a cluster, 2α the solid angle of the cone, and $f(\beta)$ the mean distribution of stars with galactic latitude. Then

$$N = \int_{\beta_0 - \alpha}^{\beta_0 + \alpha} \int_{\lambda_1(\alpha, \beta_0, \lambda_0)}^{\lambda_2(\alpha, \beta_0, \lambda_0)} f(\beta) \cos \beta \, d\beta \, d\lambda$$

or, since

$$\cos \alpha = \sin \beta \sin \beta_0 + \cos \beta \cos \beta_0 \cos (\lambda - \lambda_0),$$

we find
$$N = 2\int_{\beta_0-\alpha}^{\beta_0+\alpha} f(\beta)\cos\beta \arccos\frac{\cos\alpha - \sin\beta\sin\beta_0}{\cos\beta\cos\beta_0}\,d\beta. \quad (4)$$

The integration was performed numerically. A definite value of v, and hence a *linear* correlation between red-shift and star numbers, was found only for solid angles of 60°. The correlation also holds for larger angles, but for those much smaller than 60°, the star numbers increase too fast with decreasing latitude.

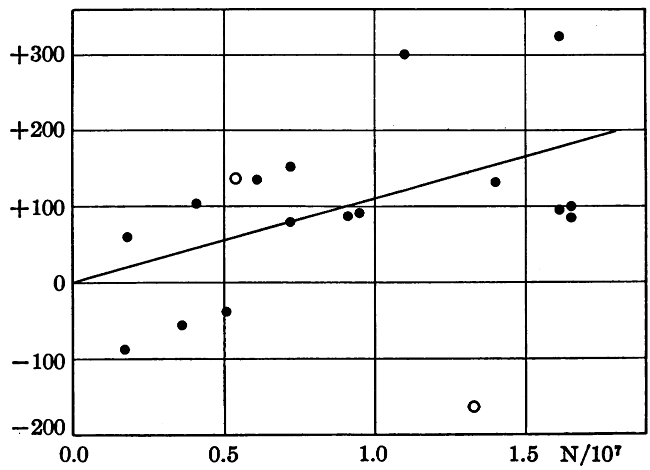

FIGURE 2

Values of $V-(K + a\xi + b\eta + c\zeta)$ plotted against star numbers (abscissae). Circles indicate uncertain values. The straight line represents the least-squares solution.

The solution for solid angles of 60° was also made without a K term. The results, derived from 16 clusters (two clusters with uncertain velocities have been excluded) are:

With K term	*Without K term*
$\xi = -41 \pm 58$ km./sec.	$\xi = -43 \pm 56$ km./sec.
$\eta = +131 \pm 63$	$\eta = +129 \pm 61$
$\zeta = -171 \pm 102$	$\zeta = -197 \pm 91$
$v = +109 \pm 79$	$v = +73 \pm 50$
$K = -37 \pm 61$	$K = 0$

The two solutions are practically identical, and give a mean red-shift of about

$$v = +91 \pm 65 \text{ km./sec.} \quad (5)$$

The negative K-term in the first solution cannot be considered as real.

The values of $V - (K + a\xi + b\eta + c\zeta)$ and of $V - (a\xi + b\eta + c\zeta)$ are plotted against the star numbers N in figures 2 and 3. In view of the uncertainty in the observed radial velocities and the roughness with which the number of stars counted represents the mean density of matter in space, the solutions satisfy the observations very well. The values for the two excluded clusters with uncertain radial velocities are plotted in the figures as open circles.

4. In interpreting the correlation (3) for the clusters on the same physical basis as for the spirals, it must be shown that the observed red-shift (5) is of the order of magnitude predicted by Zwicky's formula:

$$\frac{\Delta\nu}{\nu} = \alpha \Gamma \rho \frac{l^2}{c^2}, \tag{6}$$

where Γ is the gravitational constant, ρ the mean density of matter inside the cone, l the distance traveled by the light quanta, c the velocity of light, and α a constant of the order of unity.

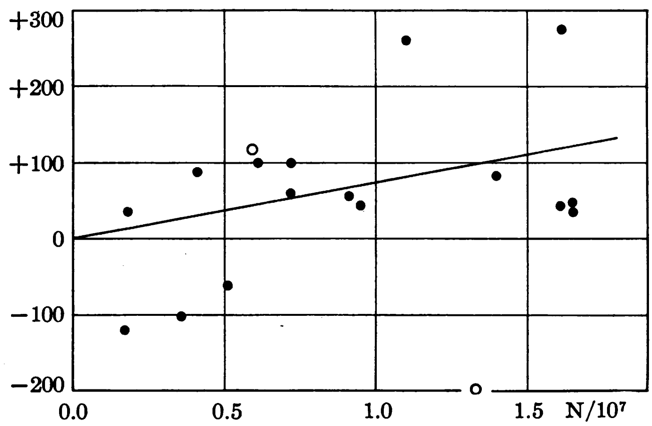

FIGURE 3

Results similar to those of Fig. 2 with the K term omitted from the solution.

From the least-squares solutions we have found $v = +0.9 \times 10^7$ cm./sec. for $N = 1.0 \times 10^7$. The counted number N is approximately proportional to the *total* number of stars inside the cone (N_t), and v is proportional to the real red-shift v_t. Hence

$$Nv = N_t v_t. \tag{7}$$

Since

$$\frac{\Delta\nu}{\nu} = \frac{v_t}{c} \text{ and } \rho = 3.8 \times 10^{33} \frac{N_t}{l^3} \text{ gr./cm.}^3,$$

which assumes that the mean mass of stars within the cone of 60° solid angle equals the mass of the sun (2×10^{33} gr.), Zwicky's formula (6) gives

$$v_t = \alpha \, \Gamma \, (3.8 \times 10^{33}) \frac{N_t}{lc} \text{ cm./sec.} \qquad (8)$$

For $N = 1.0 \times 10^7$ we have

$$N_t v_t = 3.8 \times 10^{33} \, \alpha \, \Gamma \frac{N_t^2}{lc} = 0.9 \times 10^{14}.$$

With

$l = 16{,}000$ parsecs $= 5 \times 10^{22}$ cm., $c = 3 \times 10^{10}$ cm./sec., $\Gamma = 6.7 \times 10^{-8}$,

we find

$$\alpha \, N_t^2 = 0.6 \times 10^{21}.$$

Since α is of the order of unity, this gives

$$N_t = \beta \times 10^{10}.$$

Whence

$$N_t = \beta \times 10^3 \times N, \qquad (9)$$

β being a constant of the order of unity.

This means that the total number of stars required to bring the observed value of the red-shift into agreement with Zwicky's theoretical result is 1000 times the number brighter than the eighteenth magnitude. The direct evidence bearing on this point is rather slender and depends on an uncertain extrapolation of the behavior of the stars which can be observed. The mean distribution tables of Seares and van Rhijn[8] indicate for stars in the galactic plane a value of about 500 for the above ratio. This is a mean result, averaged over all longitudes, and is not directly applicable because of the eccentric position of the sun within the galactic system. Since most of the clusters are in the general direction of the center of the stellar system, the ratio of 500 must be increased by an unknown factor, perhaps as large as two or three. Non-luminous and obscured matter will also contribute something. On the other hand, the fact that the latitudes are not zero will diminish the ratio derived from the mean distribution table. A balancing of these opposing tendencies leads to the conclusion that the theoretical ratio of 1000 to 1 found above is of the right general order of magnitude to reconcile the observed red-shift with Zwicky's prediction.

5. Our results may be summarized as follows:

(a) There exists a linear correlation between the displacements of spectral lines for globular clusters, corrected for solar motion, and galactic latitude, similar to the correlation between line displacement and distance which holds for the spirals.

(b) The correlation may express a dynamical property of the larger galactic system.

(c) The correlation may express a physical relation between red-shift and total amount of matter between us and the clusters. Proceeding on this assumption, we find a linear correlation between red-shift and star numbers in cones with solid angles of 60° or larger. The amount of the red-shift seems to be of the general order required by Zwicky's formula.

I wish to express my best thanks to Dr. Hubble and Dr. Strömberg, and especially to Profesor Seares and Dr. Zwicky, for many suggestions and much valuable information relating to the subject. Also my best thanks are due to Dr. Adams for permission to publish my results in this way.

1 Gustaf Strömberg, *Mt. Wilson Contr.*, No. 292; *Astrophys. J.*, Chicago, Ill., **61**, 1925 (353–362).

2 E. von der Pahlen and E. Freundlich, *Pub. Astrophys. Obs.*, Potsdam, **26** (No. 86), 1928.

3 Edwin Hubble, *Mt. Wilson Comm.*, No. 105; these PROCEEDINGS, **15**, 1929 (168–173).

4 M. L. Humason, *Mt. Wilson Comm.*, No. 104; these PROCEEDINGS, **15**, 1929 (167–168).

5 R. C. Tolman, *Astrophys. J.*, Chicago, Ill., **15**, 1929 (297–304).

6 F. Zwicky, these PROCEEDINGS, **15**, 1929 (773–779).

7 If the observed radial velocities are plotted directly against galactic latitude, the correlation is much worse.

8 F. H. Seares, P. J. van Rhijn, et al., *Mt. Wilson Contr.*, No. 301, Table XVII; *Astrophys. J.*, Chicago, Ill., **62**, 1925 (320–374).

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*SOLID PROTEIN HYDROCHLORIDES**

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If one adds hydrogen chloride gas at a constant temperature to a solid and if one plots the amount of hydrogen chloride taken up by unit mass of the solid as abscissas against the equilibrium pressures as ordinates, it is an easy matter to tell whether hydrogen chloride forms a definite chemical compound with the solid according to stoichiometric proportions or whether the hydrogen chloride is adsorbed by the solid. If a definite chemical compound is formed, we know from the phase rule that the equilibrium pressure must remain constant so long as there are two solid phases present, the pure solid and a compound, or two compounds. A flat in the isotherm as plotted means the existence of a second solid phase. If there is adsorption and no definite formation of a compound according to stoichiometric proportions, the isotherm will be a smooth curve bending away from the axis of abscissas at higher concentrations.